

Particle production in a gravitational wave background

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Abstract

We study the possibility that massless particles, such as photons, are produced by a gravitational wave. That such a process should occur is implied by tree-level, Feynman diagrams such as two gravitons turning into two photons *i.e.* $g + g \rightarrow \gamma + \gamma$. Here we calculate the rate at which a gravitational wave creates a massless, scalar field. This is done by placing the scalar field in the background of a plane gravitational wave and calculating the 4-current of the scalar field. Even in the vacuum limit of the scalar field it has a non-zero vacuum expectation value (similar to what occurs in the Higgs mechanism) and a non-zero current. We associate this with the production of scalar field quanta by the gravitational field. This effect has potential consequences for the attenuation of gravitational waves since the massless particles are being produced at the expense of the gravitational field. This is related to the (time-dependent) Schwinger effect but with the electric field replaced by the the gravitational wave background and the electrons/positrons replaced by massless scalar “photons”. Since the produced scalar quanta are massless there is no exponential suppression as occurs in the Schwinger effect due to the electron mass.

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I. INTRODUCTION

As early as 1855 Faraday recognized the possibility of a relationship between gravity and electricity [1] through his observation “Such results, if possible, could only be exceedingly small; but, if possible, *i.e.* if true, no terms could exaggerate the value of the relation they would establish”. More recently the potential relationship between gravity and the electromagnetic interactions has been examined in terms of individual quanta in terms of gravitons and photons [2–4] using Feynman diagrams or in terms of electromagnetic waves and gravitational waves [5–7] (*i.e.* large collections of photons and gravitons). The perturbative, Feynman diagrammatic calculations of [2–4] give transitions from gravitons to photons which are consistent with Faraday’s expectation that this effect is “exceedingly small”. For example in [2] it was found that the cross section for two gravitons going to two photons $(g + g \rightarrow \gamma + \gamma)$ ¹ is of the order $\sigma \sim 10^{-110} \text{cm}^2$ for a wave whose frequency is set by the electron rest mass $\omega \sim m_e$. For such a small cross section this process is not important even for gravitons traveling cosmological distances. (The frequencies involved in the detection by LIGO of GW150914 [9] were much lower than $\omega \sim m_e$, which would make the cross sections even smaller. The point of these estimates is that one gets a small, but non-zero result for this process.). In this paper we want to examine the production of massless quanta from a gravitational wave background. This can be viewed as a gravitational variant of the Schwinger effect where a strong, static electric field can produce electron-positron pairs [10]. In the present case the background field is that of a gravitational wave instead of a static electric field and the particles produced are massless scalar particles (which are models for photons) instead of electrons-positrons. In the usual Schwinger effect the rate of electron-positron production *i.e.*

$$\Gamma_{e+e-} = \frac{e^2 E_0^2}{4\pi^3} \exp \left[\frac{-\pi m_e^2}{e E_0} \right] \quad (1)$$

is suppressed by the well known exponential term at the end of the expression above (E_0 is the magnitude of the electric field and m_e is the electron mass). In the case studied here –

¹ Since here we have in mind to calculate how a gravitational plane wave, which is composed of many gravitons, is converted into a massless field, the gravitons would be taken as going in the forward direction and the massless field created from the gravitational wave would also be going in the forward direction as expected from energy-momentum conservation [8].

gravitational field creating massless quanta – there will be no exponential suppression since the mass is zero.

A final important point about taking the scalar field to be massless is that it has been shown [11] that a gravitational plane wave can not create a scalar field *if the scalar field is massive*. The caveat given in [11] for when it might be possible to create a scalar field from a gravitational plane wave is exactly when the scalar field is massless. This also fits in with the particle view point investigated in [8] where the decay of gravitons into other particles was investigated and from very simple kinematic arguments it was shown that graviton decay was only possible when the graviton decayed into other massless particles.

The potential significance of the process where electromagnetic radiation is produced from a gravitational wave background, is that this would lead to a weakening/attenuation of the gravitational wave, since the creation of the electromagnetic radiation would come at the expense of the gravitational wave. Thus, if the production of electromagnetic radiation via gravitational waves is significant, one would need to take this into account when using the detected amplitude of the gravitational wave to determine the characteristics of the event such as the distance to the source of the gravitational waves. For example, this attenuation would mean that the source of the gravitational wave was closer than implied by the measured amplitude. Another consequence of this process is that one might think to look for the electromagnetic radiation which was produced by the gravitational wave. In fact there is a claim [12] that the gravity wave detection by LIGO, GW150914, [9] was potentially accompanied by a γ -ray signal. Our calculations below will show that a gravitational wave might produce electromagnetic radiation, but rather than being in the γ -ray range, the electromagnetic radiation produced would have extremely long wave lengths on the order of 100s of kilometers.

Previously the question of production of electromagnetic radiation from a gravitational background was examined by two of the authors [6] using the formalism of the Unruh-DeWitt detector. The resulting particle production rate found in this way was small but not as small as indicated by the Feynman diagram calculations [2–4] for individual quanta. Based on the Unruh-Dewitt detector calculations of [6] it was possible that the production of electromagnetic radiation via a gravitational wave background might have an attenuating effect on the gravitational wave. This difference between the Feynman diagrammatic calculations of [2–4] and the Unruh-DeWitt detector calculations of [6] can be compared to the

situation that occurs when calculating the decay rate, $\Gamma_{e^+e^-}$, for the Schwinger effect. The expression for $\Gamma_{e^+e^-}$ given in (1) is non-perturbative (this can be seen by the presence of $\exp[-\frac{\text{const.}}{E_0}]$) and cannot be obtained via the perturbative method of Feynman diagrams.

In Minkowski space-time the calculation of vacuum pair production via different methods gives identical results. For example, one can calculate the Schwinger effect of e^+e^- creation from a static electric field via the Trace-Log method originally used by Schwinger or via the “scattering/tunneling” of the scalar field φ by the potential due to the uniform electric field and the results are the same (this comparison of different methods of calculating the Schwinger effect can be found in [13, 14] as well as in [15]). However, in curved space-time different methods for calculating pair production can give different results for the production rate as discussed in several papers [16–18]. As pointed out in [16] the difficulty of studying particle production in the presence of a gravitational background, which is not asymptotically flat, is that the definition of the particle production rate can depend on the method of calculation (*e.g.* using creation/annihilation operator versus using a definition of a vacuum state versus using a Feynman Green function). In [16] the vacuum-to-vacuum amplitude is calculated in the path integral approach for Friedmann-Robertson-Walker space-time. The amount by which this amplitude differs from unity is used to obtain the particle production rate and it is found that this gives a different particle production rate from the usual diagonalization of the Hamiltonian method. The method of using the deviation of the vacuum-to-vacuum transition amplitude from unity was also used in [19] to obtain the particle production rate for the Schwinger effect, but in this case, since the calculation was done in flat space-time the result was the same as other methods. In this paper, we calculate the particle production rate by calculating the conserved 4-current of our massless field in a gravitational wave background and then comparing this to the massless field in a flat space-time. The difference between these two situations – scalar field in a gravitational wave background versus scalar field in Minkowski space-time – we take as a measure of the rate of particle production. This 4-current method for calculating the production rate and/or super radiance is similar to that used in references [18, 20–22]. The 4-current method employed here can also be compared to the work of Gertsenshtein [5], who investigated the production of electromagnetic radiation when a gravitational wave encountered a region of space-time with a uniform magnetic field. In Gertsenshtein the interaction between the magnetic field and gravitational wave background produced electromagnetic radiation. Here we replace

the magnetic field by a massless scalar field.

In section II we study the solution of a massless scalar field in a gravitational wave background and use this to calculate a particle production rate. In section III we use the results of section II to estimate an attenuation length for the gravitational wave due to the production of electromagnetic radiation from the gravitational wave.

II. SCALAR FIELD IN GRAVITATIONAL WAVE BACKGROUND

We begin by placing a massless, scalar field ² in curved space-time by writing down the Klein-Gordon equation coupled to the space-time described by the metric $g_{\mu\nu}$,

$$\frac{1}{\sqrt{-|g_{\mu\nu}|}} \left(\partial_\mu g^{\mu\nu} \sqrt{-|g_{\mu\nu}|} \partial_\nu \right) \varphi = 0. \quad (2)$$

We take the gravitational wave to have the + polarization for which the metric [23] can be written as,

$$ds^2 = -dt^2 + dz^2 + f(u)^2 dx^2 + g(u)^2 dy^2. \quad (3)$$

The variables in the metric $u = z - t$, $v = z + t$ are light cone coordinates and the metric components $f(u)$ and $g(u)$ will be required to be oscillatory functions as expected for a gravitational wave background. The determinant of the metric is $|g_{\mu\nu}| = \det[g_{\mu\nu}]$ and $\sqrt{-|g_{\mu\nu}|} = fg$. Substituting this in equation (2),

$$\frac{1}{fg} \left(-\partial_t(fg)\partial_t + \frac{1}{f^2}\partial_x(fg)\partial_x + \frac{1}{g^2}\partial_y(fg)\partial_y + \partial_z(fg)\partial_z \right) \varphi = 0. \quad (4)$$

Since u is only a function of z and t the expression can be expanded,

$$\left(-\partial_t^2 - \frac{1}{fg}\partial_t(fg)\partial_t + \frac{1}{f^2}\partial_x^2 + \frac{1}{g^2}\partial_y^2 + \partial_z^2 + \frac{1}{fg}\partial_z(fg)\partial_z \right) \varphi = 0. \quad (5)$$

Applying the chain rule for the t and z derivatives, $\partial_t(fg) = -\partial_u(fg)$, $\partial_z(fg) = \partial_u(fg)$, $(\partial_z^2 - \partial_t^2) = 4\partial_u\partial_v$, $(\partial_t + \partial_z) = 2\partial_v$, $(\partial_z - \partial_t) = 2\partial_u$, and multiplying by f^2g^2 ,

² We have taken a massless scalar field, φ , as a stand-in for the electromagnetic potential, A_μ . The only difference is that we do not have to carry around the space-time index μ .

$$(4f^2g^2\partial_u\partial_v + 2fg\partial_u(fg)\partial_v + g^2\partial_x^2 + f^2\partial_y^2)\varphi = 0. \quad (6)$$

At this point we are still looking at the exact solution to the Klein-Gordon equation using the metric of equation (3). To evaluate the solution for a weak gravitational wave the linearized gravity approximation will be introduced in the terms of the metric, $f(u) = 1 + \varepsilon(ku)$, and $g(u) = 1 - \varepsilon(ku)$ and substituted into equation (6). Also note that the metric of equation (3) describes a wave propagating in the z direction and the x and y spatial directions must be physically indistinguishable. Based on the isotropy of space-time and assuming a non-thermal vacuum [6, 8, 24], we take $(\partial_y^2 - \partial_x^2)\varphi = 0$ as a property of our scalar field solution. Using this and collecting terms together equation (6) can be expressed as,

$$[4(1 - 2\varepsilon^2 + \varepsilon^4)\partial_u\partial_v - 4(1 - \varepsilon^2)\varepsilon(\partial_u\varepsilon)\partial_v + (1 + \varepsilon^2)\partial_x^2 + (1 + \varepsilon^2)\partial_y^2]\varphi = 0. \quad (7)$$

In order to evaluate the equation assume the oscillatory solution for linearized general relativity typical of gravitational waves $\varepsilon(ku) = h_+e^{iku}$ where h_+ is the dimensionless amplitude of the gravitational wave. Substituting into equation (7),

$$(4F\partial_u\partial_v - 4ikG\partial_v + H(\partial_x^2 + \partial_y^2))\varphi = 0, \quad (8)$$

where,

$$\begin{aligned} F(ku) &\equiv (1 - 2h_+^2e^{2iku} + h_+^4e^{4iku}), \\ G(ku) &\equiv (h_+^2e^{2iku} - h_+^4e^{4iku}), \\ H(ku) &\equiv (1 + h_+^2e^{2iku}). \end{aligned} \quad (9)$$

Equation (8) is separable taking $\varphi = X(x)Y(y)U(u)V(v)$ and identifying the eigenvalue equations for $X(x)$ and $Y(y)$ as,

$$\begin{aligned} \partial_x^2 X &= -k_x^2 X \rightarrow X = e^{ik_x x}, \\ \partial_y^2 Y &= -k_y^2 Y \rightarrow Y = e^{ik_y y}. \end{aligned} \quad (10)$$

Note, that the x and y direction eigenfunctions are simply free waves as is to be expected since the gravitational wave is in the $u = z - t$ direction. Setting $2k_{xy}^2 \equiv k_x^2 + k_y^2$ and using (10) we find that (8) becomes

$$F \frac{\partial_u U}{U} \frac{\partial_v V}{V} - ikG \frac{\partial_v V}{V} - H \frac{k_{xy}^2}{2} = 0. \quad (11)$$

Now since the light front coordinate v is orthogonal to u and since the gravitational wave only depends on u one expects that the eigenfunction $V(v)$ also is solved by a free, plane wave, as was the case for $X(x)$ and $Y(y)$. This is indeed the case and we find

$$-i\partial_v V = k_v V \rightarrow V = e^{ik_v v}. \quad (12)$$

Substituting equation (12) into equation (11) yields,

$$ik_v F \frac{\partial_u U}{U} + kk_v G - \frac{k_{xy}^2}{2} H = 0. \quad (13)$$

Defining the eigenvalue $\lambda \equiv \frac{k_{xy}^2}{2k_v}$ (13) can be rearranged,

$$i \frac{\partial_u U}{U} = \lambda \frac{H}{F} - k \frac{G}{F}, \quad (14)$$

and this equation can be integrated to give,

$$U = e^{\frac{\lambda}{k}} e^{\frac{-\lambda}{k(1-h_+^2 e^{2iku})}} (1 - h_+^2 e^{2iku})^{\frac{1}{2}(\frac{\lambda}{k}-1)} e^{-i\lambda u}. \quad (15)$$

The first term ($e^{\frac{\lambda}{k}}$) is needed to insure that as $h_+ \rightarrow 0$ (*i.e.* the gravitational wave is turned off) that the eigenfunction for the u direction becomes a free plane wave, $e^{-i\lambda u}$. Collecting together all the terms in x, y, v and u directions gives the solution of the scalar field in the gravitational background,

$$\varphi = e^{\frac{\lambda}{k}} e^{-\frac{\lambda}{k(1-h_+^2 e^{2iku})}} (1 - h_+^2 e^{2iku})^{\frac{1}{2}(\frac{\lambda}{k}-1)} e^{-i\lambda u} e^{ik_v v} e^{ik_x x} e^{ik_y y}. \quad (16)$$

This solution for the scalar field given in (16) is very similar to solution found in [15] for the static electric field pair production evaluated in light front coordinates.

Taking the limit $h_+ \rightarrow 0$ of equation (16) returns the expected Minkowski vacuum solution for the scalar field,

$$\varphi_0 = e^{-i\lambda u} e^{ik_v v} e^{ik_x x} e^{ik_y y}, \quad (17)$$

or in terms of the time and space coordinates,

$$\varphi_0 = e^{i(k_v+\lambda)t} e^{i(k_v-\lambda)z} e^{ik_x x} e^{ik_y y}. \quad (18)$$

It is clear that the scalar field in (18) is a free wave. By defining an energy $k_0 = k_v + \frac{k_{xy}^2}{2k_v}$ and a momentum in the z -direction $k_z = k_v - \frac{k_{xy}^2}{2k_v}$ and using the previously defined $k_x^2 + k_y^2 = 2k_{xy}^2$ one can check that energy-momentum of the free solution in (18) satisfy the usual kinematic relationship for a free particle in Minkowski space-time namely $k_0^2 = k_x^2 + k_y^2 + k_z^2$.

We now will determine the rate of pair production using the 4-current density of the scalar field in the gravitational wave background given in (16). The u component of the scalar 4-current can be calculated directly from the solution, φ , given by equation (16),

$$j_u = -i (\varphi^* \partial_u \varphi - \varphi \partial_u \varphi^*). \quad (19)$$

Substituting φ from (16) into (19) we find that the u component of the scalar field 4-current is,

$$j_u = -2\lambda - \left(\frac{9}{2} \frac{\lambda^3}{k^2} - \frac{12\lambda^2}{k} + \frac{13}{2} \lambda - k \right) h_+^4. \quad (20)$$

In obtaining this expression we have taken the light front coordinate averages for the cosines, $\langle \cos^2(2ku) \rangle = \frac{1}{2}$, $\langle \cos^4(2ku) \rangle = \frac{3}{8}$, and $\langle \cos(2ku) \rangle = \langle \cos(4ku) \rangle = 0$. Also we have dropped terms higher than $\mathcal{O}(h_+^4)$.

The result in equation (20) is a key result of this work and we want to examine various limits. First, in the limit when the gravitational wave vanishes, $h_+ \rightarrow 0$, the current becomes $j_u \rightarrow -2\lambda$ which is exactly what is expected for the current of a free particle with energy-momentum characterized by λ (see section 4.3 of [25]). Second, in the presence of both the scalar field ($\lambda \neq 0$) and gravitational wave ($h_+ \neq 0$) equation (20) indicates how the current is modified by the potential represented by the gravitational background. For certain values of λ, k , and h_+ the current in (20) gives a larger outgoing current than incoming. This can be likened to the calculation of the Penrose super radiance process [22] where one “scatters” a real scalar field from a rotating black hole and the outgoing scalar field may have more energy than the incoming field. Finally, one can take the limit $\lambda \rightarrow 0$, $k_v \rightarrow 0$ and $k_{xy} \rightarrow 0$ *i.e.* the initial scalar field is taken to its vacuum state. In this way one obtains what is called the Minkowski persistence amplitude [19]. Because of the definition $\lambda \equiv \frac{k_{xy}^2}{2k_v}$ the limit

$\lambda \rightarrow 0$ also means $k_{xy} \rightarrow 0$. In this limit the scalar field and its 4-current (16) *do not* reduce to the vacuum case (*i.e.* $\varphi_0 \rightarrow 0$ and $j_u \rightarrow 0$) but rather reduce to

$$\varphi \rightarrow (1 - h_+^2 e^{2iku})^{-\frac{1}{2}} \quad \text{and} \quad j_u \rightarrow kh_+^4. \quad (21)$$

The result in (21) can be related to the Higgs mechanism [26]. In the Higgs mechanism a scalar field develops a non-zero vacuum expectation value of $\varphi = \sqrt{\frac{m^2}{2\lambda}}$ due to a quartic *self interaction* term plus tachyonic mass term of the form $-m^2\varphi^2 + \lambda\varphi^4$. The self interacting scalar potential in the usual Higgs mechanism is time independent. In the present case the scalar field develops a non-zero vacuum expectation value (the first term in (21)) due to the background gravitational wave potential. Due to the space and time dependent nature of the background gravitational field the vacuum expectation value from (21) is also dependent on space and time – the e^{2iku} term in the expression for φ . Since the vacuum expectation value in this case is space and time dependent, one has a non-zero 4-current in the $u = z - t$ direction, $j_u = kh_+^4$, as opposed to the usual Higgs mechanism case where the vacuum expectation value of the scalar field does not vary with space and time, and there is no 4-current associated with the standard Higgs mechanism solution $\varphi = \sqrt{\frac{m^2}{2\lambda}}$. Another difference between the present example and the canonical Higgs mechanism, is that in the present example the interaction that leads to the vacuum expectation value of φ in (21) is the interaction between the scalar field and the gravitational field. In the canonical Higgs mechanism the vacuum expectation value is due to the $\lambda\phi^4$ self interaction of the scalar field. Thus the present example of the non-zero scalar vacuum expectation value can be compared to the version of the Higgs mechanism that occurs in superconductors where it is the phonons of the background lattice that are responsible for the interaction that binds electrons into Cooper pairs and which lead to superconductivity.

The expression for φ in this limit is reminiscent of the scalar field form obtained for the Schwinger effect done in light front coordinates as given in [15] (see equation 3.26 of [15]). The important point about (21) is that $j_u \neq 0$ even though we have taken the scalar field to its Minkowski vacuum state. We interpret this non-zero j_u as the particle production rate. If we restore c then (21) becomes $j_u = ckh_+^4 = \omega h_+^4$ which has units of a rate per unit volume (the volume unit comes from the scalar fields in the definition of the current in (19)). Thus we take (21) to represent the production of the massless scalar field via a gravitational wave background. That one should get a non-zero result for the process of

gravitons converting to these scalar “photons” is to be expected since the Feynman diagram amplitudes for $g + g \rightarrow \gamma + \gamma$ or $g \rightarrow g + \gamma + \gamma$ are non-zero [2–4]. It should be noted that in the above Feynman diagram amplitudes for individual quanta the gravitons and photons must all be traveling in the same direction due to energy-momentum conservation. This issue was discussed in [8] which looked at constraints on the decay of massless particles. One of the key restrictions found in [8] was that the decay products had to be massless and have their momentum along the direction of the incoming quanta.

The calculation of the production of the scalar field via the time varying gravitational wave background of (3) can be compared to the similar calculation for de Sitter space-time from reference [27]. There a massive scalar field was placed in the time-dependent de Sitter space-time and the amplitude of the scalar field in the de Sitter background was used to determine the scalar field production rate at the expense of the gravitational field. Unlike the de Sitter space-time metric there is no horizon in the gravitational wave metric and the production is associated with the time dependence of the metric, similar to cosmological expansion [28, 29].

One might ask if the linear approximation for the gravitational wave – namely that $f(u) = 1 + \epsilon(ku)$ and $g(u) = 1 - \epsilon(ku)$ with $\epsilon(ku) = h_+ e^{iku}$ – is crucial in obtaining the result in (21). What if one took an exact plane wave solution instead of a linearized approximation? To this end we now repeat briefly the above analysis for an exact, plane wave, solution for the metric (3). The condition for $f(u)$ and $g(u)$ to be exact plane wave solutions to the Einstein field equations is that they should satisfy the condition $\ddot{f}/f + \ddot{g}/g = 0$ [23]. One simple exact, plane wave, solution is $f = e^{iku} e^{-ku}$ and $g = e^{iku} e^{ku}$. These ansatz functions have plane wave parts (e^{iku}) but they also have exponentially growing or decaying amplitudes ($e^{\pm ku}$). Near $u = 0$ one has oscillating, wave solutions due to the e^{iku} parts of the ansatz function, but as u moves away from $u = 0$ the $e^{\pm ku}$ terms act like growing/decaying amplitudes. Because of this these solutions can only be of use for a restricted range of u near $u = 0$. Asymptotically, as $u \rightarrow \infty$, the functions $f(u), g(u)$ are not physically acceptable. Substituting $f = e^{iku} e^{-ku}$ and $g = e^{iku} e^{ku}$ into equation (6),

$$(4e^{4iku} \partial_u \partial_v + 2e^{2iku} \partial_u (e^{2iku}) \partial_v + e^{2iku} e^{2ku} \partial_x^2 + e^{2iku} e^{-2ku} \partial_y^2) \varphi = 0, \quad (22)$$

and making the substitution $\varphi = U(u)V(v)X(x)Y(y) = U(u)e^{ik_v v} e^{ik_x x} e^{ik_y y}$,

$$\left(i\frac{\partial_u U}{U} - k - e^{-2iku}e^{2ku}\frac{k_x^2}{4k_v} - e^{-2iku}e^{-2ku}\frac{k_y^2}{4k_v}\right) = 0. \quad (23)$$

In the limit when the gravitational wave is absent (*i.e.* $k \rightarrow 0$) the solution to (23) is again given by (18). When $k \neq 0$ the solution is (23),

$$U = e^{\left(\frac{(1-i)}{4k}\lambda_x e^{-2iku}e^{2ku} + \frac{(1+i)}{4k}\lambda_y e^{-2iku}e^{-2ku}\right)} e^{-iku}, \quad (24)$$

where $\lambda_x \equiv \frac{k_x^2}{4k_v}$ and $\lambda_y \equiv \frac{k_y^2}{4k_v}$. Here we do not have an h_+ since the changing “amplitude” is given by the $e^{\pm ku}$ terms in $f(u), g(u)$. As before if we take the limit of the massless scalar field to its vacuum state (*i.e.* taking the limit $k_x \rightarrow 0$, $k_y \rightarrow 0$, $\lambda_{x,y} \rightarrow 0$ and $k_v \rightarrow 0$, one finds $U(u) \rightarrow e^{-iku}$ so that as before φ does not go to zero but rather $\varphi \rightarrow e^{-iku}$). As before we can calculate the current in the u direction in this limit and find that,

$$j_u = \lim_{(k_x, k_y) \rightarrow 0} -i(U^* \partial_u U - U \partial_u U^*) = 2k. \quad (25)$$

As in (21) we take (25) to be a measure of the production rate per unit volume of the scalar field and find that the decay rate is $\propto k \rightarrow ck = \omega$. There is no explicit amplitude, h_+ , in this case since the changing amplitudes of the ansatz functions, $f(u), g(u)$, are given by $e^{\pm ku}$.

The estimate of the distance to the source for GW150914 was based on the shape of the detected signal and the strain amplitude which was assumed to depend only on the usual $\frac{1}{r}$ fall off with a spherical wave. If the decay of the gravitational wave, due to conversion into massless scalar “photons” via the above mechanism, is significant, one would have to take into account this decay in h_+ plus the $\frac{1}{r}$ fall off. If the fall off of h_+ from the decay rate implied in (21) is significant then one might over estimate the distance between the LIGO detectors and the source of the gravitational waves of GW150914 – two inspiraling black holes. In the next section we make some rough estimates of this decay length.

III. ESTIMATED ATTENUATION LENGTH

The scalar field production implied by the current in (21) can be compared to the perturbative results of $g + g \rightarrow \gamma + \gamma$ of [2] where Feynman diagram methods were used. In [2] it was found that the cross section for $g + g \rightarrow \gamma + \gamma$ went like κ^4 where $\kappa = \sqrt{16\pi G}$, and in

(21) the production rate depends on the fourth power of the amplitude, h_+^4 . Other, recent works which connect the particle production rate with currents in curved space-times, in a manner similar to what we used to obtain (21), can be found in [18, 20, 21]. To make an estimate of the effect this has on the gravitational wave, we use the production rate for the massless scalar field from (21) to give a decay rate for the gravitational wave of the form

$$\Gamma = kh_+^4 \rightarrow \omega h_+^4, \quad (26)$$

where we have restored a factor of c , so that $k \rightarrow ck = \omega$ and then the dimensions of Γ are explicitly those of a rate per unit volume. The per unit volume comes from the dimensionality of φ used in calculating the 4-current and the per unit volume is still implicit in (26).

In a naive way we can turn Γ from (26) into a decay length as follows:

$$\Lambda = \frac{c}{\Gamma} = \frac{c}{\omega h_+^4}. \quad (27)$$

The relation between the production rate and decay length in equations (26) and (27) assumes that h_+ remains fixed. This is not the case since h_+ decreases due to the production of the scalar field φ at the expense of the gravitational wave. Nevertheless, if we use (27) to estimate the decay length for various h_+ 's (using $\omega \sim 3 \times 10^2 Hz$ which is roughly the frequency of the first detected gravitational wave by the LIGO collaboration [9]) we find

h_+	Λ
10^{-21}	$10^{90}m$
10^{-15}	$10^{66}m$
10^{-9}	$10^{42}m$
10^{-5}	$10^{26}m$
10^{-3}	$10^{18}m$

TABLE I: Various values of the decay length Λ versus h_+ for $\omega \approx 3 \times 10^2 Hz$. We begin with $h_+ \approx 10^{-21}$ which is roughly the measured strain reported by LIGO for GW150914 [9].

The size of the observable Universe is approximately 10^{27} meters so from Table I we see that the estimated decay length will be important, even on cosmological distances, only if

h_+ is fairly large – of the order 10^{-5} or larger. But we stress again that the Λ 's from Table I assume that the decay rate $\Gamma \propto h_+^4$ is constant which is not the case – h_+ will decrease both via the usual $\frac{1}{r}$ fall off associated with a spherical wave and the conversion of the gravitational field into the massless scalar field. This will decrease Γ (increase Λ) significantly once one moves away from the source. The take away message from Table I is that the conversion of the gravitational background wave into the massless field is significant only close to the source of the gravitational waves where h_+ is large. The results of Table I are consistent with the perturbative, tree-level Feynman diagram calculations of $g + g \rightarrow \gamma + \gamma$ from [2] – that these processes are generally not important, even at cosmological distances, unless $h_+ > 10^{-5}$.

Based on the above discussion let us try to take into account the effect of the decrease in Γ due to the decrease in h_+ coming *only* from the conversion of the gravitational wave into the scalar field. We ignore the effect of the usual $\frac{1}{r}$ fall off in h_+ due to spherical nature of the outgoing gravitational wave. The estimate for the changing decay rate due to the reduction in h_+ starts with the usual differential decay rate relationship,

$$\frac{dN_g}{dt} = -\Gamma N_g \rightarrow \frac{dN_g}{dz} = -\Gamma N_g , \quad (28)$$

where N_g is number of gravitons. In anticipation that we will be more interested in a decay length than a decay time we have taken $dt \rightarrow dz/c = dz$ in the last step and set $c = 1$. As a starting assumption we will take the number of gravitons as $N_g \propto h_+^2$, which is motivated by a similar relationship to QED where the number of photons is related to the square of the vector potential, $N_\gamma \propto A_\mu A^\mu$. In this way, and using the decay rate from (26), equation (28) becomes

$$\frac{d(h_+^2)}{dz} = -\omega h_+^4 (h_+^2) \rightarrow \frac{dh_+}{dz} = -\frac{1}{2}\omega h_+^5 , \quad (29)$$

which has the solution

$$h_+(z) = (2\omega z + K_0)^{-1/4} , \quad (30)$$

where $K_0 = (h_+^{(0)})^{-4}$ and $h_+^{(0)}$ is the value of h_+ at $z = 0$. What (30) shows is that for large distances (*i.e.* large z) that h_+ falls off like $\propto z^{-1/4}$ which is slower than the $z^{-1} \sim r^{-1}$ fall off due to the spherical nature of the outgoing gravitational wave. Thus the main factor in determining the fall off of h_+ at *large distances* is just the usual $\frac{1}{r}$ fall off. However near the source of the gravitational wave, $z = 0$, the fall off in h_+ due to the conversion of the

gravitational wave field into the massless field could be important. To do more than the rough estimate leading to $h_+(z)$ in (30) we would need to take into account the fall off in h_+ due to the usual r^{-1} fall off (which should be important at all distances) in combination with the $r^{-1/4}$ fall off coming for the decay of the gravitational wave, which should be important only relatively near the source of the gravitational waves, *i.e.* near $z = 0$.

IV. DISCUSSION AND CONCLUSIONS

In this paper we have looked at the possibility that a gravitational wave background could create massless scalar particles/fields. The massless scalar particles were taken as a simplified model of a photon. This is similar to the Schwinger effect but with the static electric field replaced by a gravitational wave background and the electron/positron replaced by massless scalar “photons”. Since the created field was in this case massless, we do not expect the usual exponential suppression of the particle production rate which one finds in the Schwinger effect. Because of this lack of exponential suppression one expects this effect to potentially play a more prominent physical role. In particular we suggested that the creation of photons at the expense of the gravitational wave field would lead to an additional fall off of the dimensionless amplitude h_+ with distance from the source, on top of the usual $\frac{1}{r}$ fall off. Based on the production rate per unit volume given in (26) we first made a naive estimate of the decay length for various amplitudes h_+ . These are given in Table I. Unless $h_+ > 10^{-5}$ our estimate for the decay length, Λ , given in Table I, was so large that one would not expect this process to attenuate or weaken the gravitational wave even over cosmological distances. This was in agreement with the conclusions based on Feynman diagram calculations [2]. But close enough to the source one will have $h_+ \geq 10^{-3}$ so in this *near* region one might expect the attenuation to be important. The estimates for Λ in Table I neglect back reaction – since h_+ should fall off due to the process described here the decay rate, Γ , will change as h_+ changes. To take this changing of Γ , due to the changing of h_+ coming from photon production, into account we did the calculation in equations (28) – (30) and found that the amplitude would fall off with distance like $h_+(z) \propto z^{-1/4}$ (as opposed to a fall off of $e^{-\Gamma z}$ if $\Gamma = \text{const.}$). Since we used a gravitational plane wave this ignored the $\frac{1}{r}$ fall off for a more realistic spherical wave. The overall conclusion, both from Table I and from (30), was that the production of the massless field, φ , coming from the

gravitational wave background and the subsequent decay/attenuation of the gravitational wave background would only be important near the source of the gravitational wave.

The most open ended point of our analysis was the calculation of the particle production rate. Different methods of calculating the particle production rates for processes in *flat* space-times, such as the production electrons/positrons from a uniform electric field, give the same results. The same is not always true for curved space-times and gravity induced particle production. This point was noted in [16] where the particle production rate for a Friedmann-Robertson-Walker space-time was calculated using the difference between the vacuum-to-vacuum transition and unity, and this was shown to give a different production rate compared to the usual diagonalization of the Hamiltonian. Also in [18] it was shown that the calculation of particle production rates due to the gravitational field could give different results depending on the calculation method used.

In this work we calculated the decay rate, Γ , in (26) which was based on the comparison of the covariant 4-current of φ both in Minkowski space-time and in the gravitational wave background. The solution for the massless scalar field φ in the gravitational wave background was given in (16). Taking this solution for φ and calculating the 4-current in the u direction gave (20). Taking φ to its vacuum limit ($k_x \rightarrow 0$, $k_y \rightarrow 0$, $\lambda_{x,y} \rightarrow 0$ and $k_v \rightarrow 0$) and time-averaging gave $j_u \rightarrow kh_+^4$ which we interpreted to be the production rate per unit volume of φ from the gravitational background. Using the 4-current for calculating the particle production rate is similar to the approach in [22] for calculating the super radiance of a black hole. Other works [20, 21] have used the 4-current to determine a gravitationally induced particle production rate. The main support for the process of a gravitational wave creating photons comes from the fact that at tree-level Feynman diagram calculations indicate that processes such as $g + g \rightarrow \gamma + \gamma$ occur. In this regard we also note that in de Sitter space-time the production rate of massive scalar particles [27] is non-zero at tree-level. Finally, the Unruh-DeWitt detector calculation of reference [6] also indicates that the conversion of a gravitational wave background into a massless field occurs.

The fact that as $k_x \rightarrow 0$, $k_y \rightarrow 0$, $\lambda_{x,y} \rightarrow 0$ and $k_v \rightarrow 0$ the scalar field and 4-current take the non-unitary and non-zero limits $\varphi \rightarrow (1 - h_+^2 e^{2iku})^{-\frac{1}{2}}$ and $j_u \rightarrow kh_+^4$ can be likened to a time-dependent, Higgs-like mechanism where the scalar field develops a non-zero vacuum expectation value. The difference from the usual Higgs mechanism is that, here the effect is driven by the interaction of the scalar field with a gravitational background instead of

with a self interaction (*i.e.* $\lambda\phi^4$). Also here the vacuum value of the scalar field is space-time dependent. This Higgs-like mechanism via the gravitational background can be compared to the symmetry breaking that occurs in superconductors, where it is the background lattice and phonons which provide the mechanism leading to a non-zero expectation value for Cooper pairs. This connection to the Higgs mechanism will be discussed further in an upcoming paper [30].

Finally there are two predictions that would occur if the production of photons from the gravitational wave background were significant. First, the amplitude h_+ measured by a detector on Earth would be smaller due to the fact that this amplitude would decrease not only from the $\frac{1}{r}$ fall off for an outward traveling wave, but also the amplitude would decrease as $r^{-1/4}$ due to the production of photons from the gravitational wave background. From the $r^{-1/4}$ dependence of the particle production rate one can see that this effect would only be important relatively close to the source. Second, the gravitational wave would produce electromagnetic radiation/photons traveling in the same direction as the initial gravitational wave. In fact it has already been suggested [12] that a γ -ray signal which was detected in the same time frame as the gravitational wave signal, might be related to the gravitational wave. If the production mechanism of electromagnetic radiation from the gravitational wave background proposed here occurs and is significant, then we would predict that the gravitational wave signal should also be accompanied by an electromagnetic signal. However, in our process this electromagnetic signal should have roughly the same frequency as that of the gravitational wave. Thus we would predict that the electromagnetic wave coming from the gravitational wave would have extremely long wavelengths, on the order of 100s of kilometers *i.e.* the associated electromagnetic wave would have very large wavelengths. These wavelengths are of such a length that they could easily have gone undetected up to now.

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